

## 3-5 Videos Guide

### 3-5a

- The Chain Rule

- For  $z = f(x, y)$ , where  $x = x(t)$  and  $y = y(t)$ ,  $\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$

- Exercise:

Use the Chain Rule to find  $dz/dt$ .

$$z = \frac{x-y}{x+2y}, \quad x = e^{\pi t}, \quad y = e^{-\pi t}$$

### 3-5b

- For  $z = f(x, y)$ , where  $x = x(s, t)$  and  $y = y(s, t)$ ,

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} \quad \text{and} \quad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}$$

Exercises:

- Use the Chain Rule to find  $\partial z/\partial s$  and  $\partial z/\partial t$ .  
 $z = \tan^{-1}(x^2 + y^2), \quad x = s \ln t, \quad y = te^s$

### 3-5c

- Use the Chain Rule to find the indicated partial derivatives.

$$T = \frac{v}{2u+v}, \quad u = pq\sqrt{r}, \quad v = p\sqrt{qr};$$

$$\frac{\partial T}{\partial p}, \frac{\partial T}{\partial q}, \frac{\partial T}{\partial r} \quad \text{when } p = 2, q = 1, r = 4$$

### 3-5d

- Higher-order derivatives

### 3-5e

- Implicit differentiation

- If  $F(x, y) = 0$ , then  $\frac{dy}{dx} = -\frac{F_x}{F_y}$ , provided  $F_y \neq 0$

- If  $F(x, y, z) = 0$ , then  $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$  and  $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$ , provided  $F_z \neq 0$

Exercise:

- Use the above formula to find  $\partial z/\partial x$  and  $\partial z/\partial y$  for  $x^2 - y^2 + z^2 - 2z = 4$ .